CALCULATION OF THE TURBULENT CHARACTERISTICS OF CHANNEL FLOW WITH ROTATING INNER CYLINDER

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An attempt to analyze the turbulent characteristics of flow in an annular channel with rotating inner cylinder is described which is based on the use of the pulsating-energy-balance equations in various directions of motion. The analytical results are compared with the experimental mean velocity distribution, the pulsation intensity, the correlation, and other data.

Turbulent flow in channels between two coaxial rotating cylinders was analyzed analytically in a number of papers. Most of them are based on the assumption of small channel curvature [1, 2], while in others [3-5] the curvature is taken into account by various approximate means. On the basis of these papers, the mean velocity distribution in the flow and the shear stress can be calculated with satisfactory accuracy.

More complete information can be obtained by means of the conventional equations of mean motion in combination with the pulsating-energy-balance equations (for the total energy and the energy of the individual velocity components). Analysis of these additional equations yields, in particular, data on the distribution not only of the mean characteristics but also the pulsation characteristics in the flow.

1. Basic Equations. Let us examine the two-dimensional annular incompressible turbulent flow, statistically homogeneous on cylindrical surfaces of constant radius. For such a flow, the following relations are fulfilled:

$$v_x = v_x', \quad v_r = v_r', \quad v_\varphi = \langle v_\varphi \rangle + v_\varphi', \quad p = \langle p \rangle + p'$$
$$\langle v_x \rangle = \langle v_r \rangle = 0, \quad \langle v \rangle_\varphi = \langle v_\varphi (r) \rangle$$
$$\langle p \rangle = \langle p (r) \rangle$$

while the derivatives of the quantities averaged over the x and φ coordinates are zero. Here, $\langle v_x \rangle$, $\langle v_r \rangle$, $\langle v_\varphi \rangle$, $\langle p \rangle$, v_x', v_r', v_φ', p' are the averaged and pulsation values of the axial, radial, and tangential velocity-vector and static-pressure components; $\langle\ \rangle$ is the sign for averaging with respect to time (after Reynolds).

With allowance for these relations, the system of differential equations for the components of the correlation tensor $\langle v_i' v_j' \rangle$ has the form

$$\left\langle \frac{1}{\rho} p' \frac{\partial v_x'}{\partial x} \right\rangle - \nu \left[\left\langle \left(\frac{\partial v_x'}{\partial x} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_x'}{\partial r} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_x'}{r \partial \varphi} \right)^2 \right\rangle \right] = 0 \tag{1.1}$$

$$2 \langle v_r' v_\varphi' \rangle \frac{\langle v_\varphi \rangle}{r} + \left\langle \frac{1}{\rho} p' \frac{\partial v_r'}{\partial r} \right\rangle - \nu \left[\left\langle \left(\frac{\partial v_r'}{\partial x} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_r'}{\partial r} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_r'}{r \partial \varphi} \right)^2 \right\rangle \right] = 0 \tag{1.2}$$

$$- \langle v_r' v_\varphi' \rangle \frac{1}{r} \frac{d}{dr} (\langle v_\varphi \rangle r) + \left\langle \frac{1}{\rho} p' \frac{\partial v_\varphi'}{r \partial \varphi} \right\rangle - \nu \left[\left\langle \left(\frac{\partial v_\varphi'}{\partial x} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_\varphi'}{\partial r} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_\varphi'}{r \partial \varphi} \right)^2 \right\rangle \right] = 0 \tag{1.3}$$

$$\langle v_x' v_\varphi' \rangle \frac{2 \langle v_\varphi \rangle}{r} + \left\langle \frac{1}{\rho} p' \left(\frac{\partial v_r'}{\partial x} + \frac{\partial v_x'}{\partial r} \right) \right\rangle - 2\nu \left[\left\langle \frac{\partial v_x'}{\partial x} \frac{\partial v_r'}{\partial x} \right\rangle + \left\langle \frac{\partial v_x'}{\partial r} \frac{\partial v_r'}{\partial r} \right\rangle + \left\langle \frac{\partial v_x'}{\partial r \varphi} \frac{\partial v_r'}{r \partial \varphi} \right\rangle \right] = 0 \tag{1.4}$$

$$- \langle v_x' v_r' \rangle \frac{1}{r} \frac{d}{dr} (\langle v_\varphi \rangle r) + \left\langle \frac{1}{\rho} p' \left(\frac{\partial v_\varphi'}{\partial x} + \frac{\partial v_x'}{r \partial \varphi} \right) \right\rangle - 2\nu \left[\left\langle \frac{\partial v_x'}{\partial x} \frac{\partial v_\varphi'}{\partial x} \right\rangle + \left\langle \frac{\partial v_x'}{\partial r} \frac{\partial v_\varphi'}{\partial r} \right\rangle + \left\langle \frac{\partial v_x'}{r \partial \varphi} \frac{\partial v_\varphi'}{r \partial \varphi} \right\rangle \right] = 0 \tag{1.5}$$

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$$-\langle v_r'^2\rangle \frac{1}{r}\frac{d}{dr}(\langle v_\varphi\rangle r)+\langle v_\varphi'^2\rangle \frac{2\langle v_\varphi\rangle}{r}+\left\langle \frac{1}{\rho} p'\left(\frac{\partial v_\varphi'}{\partial r}+\frac{\partial v_r'}{r\partial\varphi}\right)\right\rangle-2\nu\left[\left\langle \frac{\partial v_r'}{\partial x}\frac{\partial v_\varphi'}{\partial x}\right\rangle+\left\langle \frac{\partial v_r'}{\partial r}\frac{\partial v_\varphi'}{\partial r}\right\rangle+\left\langle \frac{\partial v_r'}{r\partial\varphi}\frac{\partial v_\varphi'}{r\partial\varphi}\right\rangle\right]=0 \quad (1.6)$$

In the system of equations (1.1)-(1.6), the terms associated with convective turbulent-energy transport by mean motion and with its viscous and turbulent diffusion are omitted owing to their smallness [6-8].

In accordance with [6], we make use of the approximate semiempirical relations for the dissipation of pulsating motion

$$D=2\nu\sum_{k=1}^{3}\left\langle \frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}\right\rangle=\nu c_1\frac{\langle v_i'v_j'\rangle}{l^2}+\delta_{ij}\frac{2c}{3}\frac{E^{3/2}}{l} \quad (1.7)$$

and for the energy exchange between the various pulsation components

$$\frac{1}{\rho}\left\langle p'\left(\frac{\partial v_i'}{\partial x_j}+\frac{\partial v_j'}{\partial x_i}\right)\right\rangle=-k\frac{\sqrt{E}}{l}\left(\langle v_i'v_j'\rangle-\delta_{ij}\frac{2}{3}E\right),\quad E=\sum_{i=1}^{3}\frac{\langle v_i'^2\rangle}{2} \quad (1.8)$$

Here E is the kinetic energy of pulsations; l is the scale of turbulence; c, c_1, and k are empirical constants; δ_{ij} is the Kronecker delta; and i, j = 1, 2, 3.

Substituting the sum of the first three equations for the first equation in (1.1)-(1.6), and the sum of the second and third equation for the second equation, we transform this system with allowance for equalities (1.7) and (1.8) and the designations for the local dimensionless numbers*

$$R_l=\frac{l^2}{\nu}\frac{d\langle v_\varphi\rangle}{dr},\quad R_{\omega l}=\frac{l^2}{\nu}\frac{\langle v_\varphi\rangle}{r},\quad R_E=\frac{l}{\nu}\sqrt{E}$$

to the dimensionless form

$$\frac{\langle v_r'v_\varphi'\rangle}{E}R_l-\frac{\langle v_r'v_\varphi'\rangle}{E}R_{\omega l}+CR_E+C_1=0 \quad (1.9)$$

$$2\frac{\langle v_r'v_\varphi'\rangle}{E}R_l-2\frac{\langle v_r'v_\varphi'\rangle}{E}R_{wl}+\frac{(\langle v_r'^2\rangle+\langle v_\varphi'^2\rangle)}{E}(kR_E+C_1)-\frac{4}{3}(k-c)R_E=0 \quad (1.10)$$

$$2\frac{\langle v_r'v_\varphi'\rangle}{E}R_l+2\frac{\langle v_r'v_\varphi'\rangle}{E}R_{wl}+\frac{\langle v_\varphi'^2\rangle}{E}(kR_E+c_1)-\frac{2}{3}(k-c)R_E=0 \quad (1.11)$$

$$\frac{\langle v_x'v_r'\rangle}{E}(kR_E+c_1)-2\frac{\langle v_x'v_\varphi'\rangle}{E}R_{wl}-0 \quad (1.12)$$

$$\frac{\langle v_x'v_r'\rangle}{E}(R_l+R_{wl})+\frac{\langle v_x'v_\varphi'\rangle}{E}(kR_E+c_1)=0 \quad (1.13)$$

$$\frac{\langle v_r'v_\varphi'\rangle}{E}(kR_E+c_1)+\frac{\langle v_r'^2\rangle}{E}(R_l+R_{wl})-\frac{\langle v'^2\rangle}{E}2R_{wl}=0 \quad (1.14)$$

It should be noted that Eq. (1.9) is the total energy-balance equation of flow turbulence. In the approximation under consideration, it follows from this equation that energy generation and dissipation play the principal role in the turbulent energy balance. The quantities are approximately alike in the greater portion of the channel cross section, so that turbulence is almost in the state of energy equilibrium.

System (1.9)-(1.14) consists of six equations and contains eight unknowns ($\langle v_i'v_j'\rangle$, $\langle v_\varphi\rangle$, and l). In view of this, the system must be extended to include the equation for the mean flow

*The dimensionless number

$$Re=\frac{l^2}{\nu}\frac{d\langle v_\varphi\rangle}{dr}$$

is analogous to local Reynolds number

$$R=\frac{l^2}{\nu}\frac{d\langle v_x\rangle}{dy}$$

first introduced by Loitsyanskii [9].

$$\frac{\partial}{\partial r}(\langle v_r' v_\varphi' \rangle r^2) = \nu \frac{\partial}{\partial r}\left[r^3 \frac{\partial(\langle v_\varphi \rangle / r)}{\partial r}\right]$$

which after single integration and utilization of the designations previously introduced takes the form

$$R_{wl} - R_l + \frac{\langle v_r' v_\varphi' \rangle}{E} R_E^2 = \left(\frac{v_{*i} l}{\nu}\right)^2 \left(\frac{r_i}{r}\right)^2 \tag{1.15}$$

Furthermore, it is necessary to determine the scale of turbulence l.

By solving the system of equations (1.9)-(1.14), it is not difficult to obtain

$$\langle v_\varphi'^2 \rangle / E = \frac{{}^2/_3 (k - c) R_E}{(kR_E + C_1)} - \frac{2(cR_E + c_1)(R_l + R_{wl})}{(R_{wl} - R_l)(kR_E + c_1)} \tag{1.16}$$

$$\langle v_r'^2 \rangle / E = \frac{{}^4/_3 (k - c) R_E R_{wl}}{(kR_E + c_1)(R_l + R_{\omega l})} + \frac{4(cR_E + c_1) R_{wl}}{(R_l - R_{w.})(kR_E + c_1)} - \frac{(CR_E + c_1)(kR_E + c_1)}{(R_{wl} - R_l)(R_{wl} + R_l)} \tag{1.17}$$

$$\frac{\langle v_x'^2 \rangle}{E} = 2 - \frac{\langle v_r'^2 \rangle}{E} - \frac{\langle v_\varphi'^2 \rangle}{E} \tag{1.18}$$

$$\langle v_x' v_r' \rangle = \langle v_x' v_\varphi' \rangle = 0, \quad \frac{\langle v_r' v_\varphi' \rangle}{E} = \frac{CR_E + c_1}{R_{wl} - R_l} \tag{1.19}$$

$$\frac{{}^2/_3 (k - c) R_E (R_{\omega l} - R_l)^2}{8R_{wl}(R_l + R_{wl}) + (kR_E + c_1)^2} = cR_E + c_1 \tag{1.20}$$

The left- and right-hand sides of Eq. (1.20) represent in dimensionless form the generation and dissipation of pulsation energy.

2. Flow Region at the Channel Walls. The inequality $\langle v_\varphi \rangle / r \ll d\langle v_\varphi \rangle / dr$ holds at the channel walls, and consequently $R_{wl} \gg R_l$. Making use of these inequalities, we transform relations (1.16-1.20) to the form

$$v_{\varphi i}^* = \left[\frac{{}^2/_3 (k/c - 1) R_E}{(k/c) R_E + c_1/c} + \frac{2(R_E + c_1/c)}{(k/c) R_E + c_1/c}\right]^{1/2} \frac{R_E}{\eta_i^+} \tag{2.1}$$

$$v_{ri}^* = \left[\frac{{}^2/_3 (k/c - 1) R_E}{(k/c) R_E + c_1/c}\right]^{1/2} \frac{R_E}{\eta_i^+} \tag{2.2}$$

$$v_{xi}^* = \left[2 - \frac{{}^4/_3 (k/c - 1) R_E}{(k/c) R_E + c_1/c} - \frac{2(R_E + c_1/c)}{(k/c) R_E + c_1/c}\right]^{1/2} \frac{R_E}{\eta_i^+} \tag{2.3}$$

$$\langle v_x' v_r \rangle = \langle v_x' v_\varphi' \rangle = 0, \quad v_i^{**} = -\frac{CR_E + c_1}{R_l} \frac{R_E^2}{(\eta_i^+)^2} \tag{2.4}$$

$$\frac{{}^2/_3 (k - c) R_E R_l^2}{(kR_E + c_1)^2} = cR_E + c_1 \tag{2.5}$$

Let us also express through R_E, R_e, and η_i^+ the quantities

$$E^* = \frac{R_E}{\eta_i^+}, \quad \frac{d\varphi}{d\eta_i} = (-1)^{i+1} \frac{R_l}{(\eta_i^+)^2} \quad (i = 1, 2) \tag{2.6}$$

Here, $\varphi = \langle v_\varphi \rangle / v_{*i}$; $v_{*i} = \sqrt{\tau_i / \rho}$ is the dynamic velocity, τ_1 and τ_2 are the shear stress at the rotating and stationary channel walls, respectively

$$v^*_{ij} = \frac{\sqrt{\langle v_j'^2 \rangle}}{v_{*i}} \quad (j = \varphi, r, x); \quad v_i^{**} = \frac{\langle v_r' v_\varphi' \rangle}{v_{*i}^2}$$

$$E^* = \frac{\sqrt{E}}{v_{*i}}, \quad \eta_i^+ = \frac{l v_{*i}}{\nu}, \quad \eta_i = \frac{y v_{*i}}{\nu}$$

Assuming that

$$\langle v_r' v_\varphi' \rangle = -\varepsilon \frac{d\langle v_\varphi \rangle}{dy} \tag{2.7}$$

(ε is the turbulent kinematic viscosity coefficient), and taking Eq. (2.4) into account, we obtain

$$\frac{\varepsilon}{\nu} = \frac{(cR_E + c_1) R_E^2}{R_l^2} \tag{2.8}$$

Neglecting the term associated with the influence of physical viscosity ($c_1 = 0$) in equality (2.4), and making use of relation (2.5), the correlation $\langle v_r', v_\varphi' \rangle$ in the region of fully developed turbulent flow at the walls may be written as

$$-\rho \langle v_r' v_\varphi' \rangle = \frac{[{}^2/_3 (k/c - 1)]^{3/2}}{k^3/c} \rho l^2 \left| \frac{d\langle v_\varphi \rangle}{dr} \right| \frac{d\langle v_\varphi \rangle}{dr} \tag{2.9}$$

Since in relations (1.7) and (1.8) the scale of turbulence l is defined with an accuracy to within a constant factor, from the selection of which depend the values of the constants k, c, and c_1, we set [8]

$$\frac{{}^2/_3 (k/c - 1)^{3/2}}{k^3/c} = 1 \tag{2.10}$$

In this case equality (2.9) takes the form of the well-known Prandtl formula; according to Prandtl, one may also set $l = \varkappa y$, where $\varkappa = 0.4$. With the aid of relation (2.10), the coefficients k and c may be replaced by their ratio k/c

$$k^2 = \frac{[{}^2/_3 (k/c - 1)]^{3/2}}{k/c}, \qquad c^2 = \frac{[{}^2/_3 (k/c - 1)]^{3/2}}{(k/c)^3}$$

thereby reducing the number of empirical constants.

To calculate the principal characteristics of annular turbulent flow, it is necessary to establish the dependence of R_E, R_e and η_i^+ on the η_i-coordinate. Here, the relationship between the numbers R_E and R_e, obtained from formula (2.5) with allowance for (2.10), has the form

$$R_l = -c^{1/3} \left(R_E - \frac{c_1}{k} \right) \left[\frac{R_E + c_1/c}{R_E} \right]^{1/2} \tag{2.11}$$

The results of calculations from formula (2.11) for values of the constants $k/c = 7$ and $c_1 = 2.5$ (selected as in [8], which deals with the flow in a rectilinear tube) are shown in Fig. 1 (curves I and II).

Substituting $\langle v_r' v_\varphi' \rangle$ from (2.4) into Eq. (1.15) and taking into account relation (2.11), we transform (1.15) as follows:

$$R_l + \frac{R_E^2 (CR_E + c_1)}{R_l} = -\varkappa^2 \eta_i^2 \tag{2.12}$$

Since the relationship between R_e and R_E is known, we can represent the left-hand side of Eq. (2.12) as a function of R_e

$$R_l + \frac{R_E^2 (cR_E + c_1)}{R_l} = F(R_l) \tag{2.13}$$

which is shown in Fig. 1 (curve III). Then, having found from the relation

$$-\varkappa^2 \eta_i^2 = F(R_e) \tag{2.14}$$

the distribution of the dimensionless number R_l over the channel cross section, it is not difficult to calculate from formulas (2.1), (2.8) the remaining characteristics of the flow, including the velocity profile at the wall.

3. Turbulent Flow Core. In the turbulent flow core, where the motion is close to potential rotation [5] governed by the law

$$\langle v_\varphi \rangle r = c_0 v_1 r_1 = \mathrm{const} \quad (c_0 \approx 0.55) \tag{3.1}$$

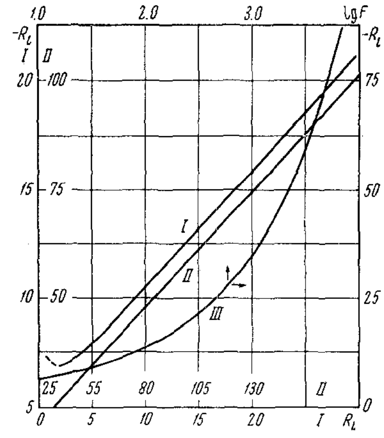

Fig. 1. R_e number as a function of R_E (curves I and II) and F (curve III).

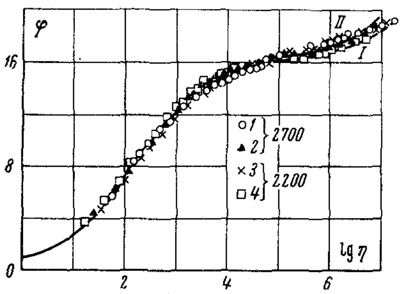

Fig. 2. Generalized dimensionless velocity profile $\varphi(\eta)$: the solid curves are reference curves; I and II refer to calculations from formula (3.18) for i = 1 and 2, respectively; 1 and 3 refer to measurements at the rotating wall; 2 and 4 refer to measurements at the stationary wall.

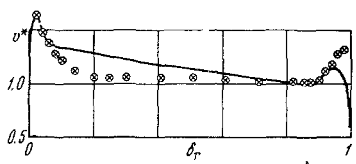

Fig. 3. Distribution of the pulsation intensity of tangential velocity component over the channel cross section.

(v_1 is the rotational speed of the cylinder), the mean vorticity of the flow $1/r\ (d < v_\varphi > r/dr)$ is very low, and consequently we may set

$$R_l + R_{\omega l} = \frac{l^2}{\nu}\frac{1}{r}\frac{d}{dr}(\langle v_\varphi\rangle r) \approx 0 \tag{3.2}$$

On the other hand, in the turbulent flow core, the terms associated with the action of viscosity forces ($c_1 \approx 0$) may be neglected. In this case, relations (1.16) (1.20) may be written in the form

$$v_{\varphi i}{}^* = k^{1/2} c^{1/6} \frac{R_E}{\eta_i{}^+} \tag{3.3}$$

$$v_{ri}{}^* = \left[kc^{1/3} - \frac{4c^{2/3} R_{\omega l}}{kR_E}\right]^{1/2} \frac{R_E}{\eta_i{}^+} \tag{3.4}$$

$$v_{xi}{}^* = \left[2 - 2kc^{1/3} + \frac{4c^{2/3} R_{\omega l}}{kR_E}\right]^{1/2} \frac{R_E}{\eta_i{}^+} \tag{3.5}$$

$$\langle v_x' v_r'\rangle = \langle v_x' v_\varphi'\rangle = 0, \qquad v_i{}^{**} = c^{2/3} \frac{R_E{}^2}{(\eta_i{}^+)^2} \tag{3.6}$$

$$R_{\omega l} - R_l = c^{1/3} R_E \tag{3.7}$$

$$E^* = \frac{R_E}{\eta_i{}^+} \tag{3.8}$$

After substitution of $< v_r' v_\varphi' >$ and R_E from formulas (3.6) and (3.7), the equation of mean motion (1.15) takes the form

$$c^{1/3} R_E = \frac{\eta_i{}^+}{r/r_i} \tag{3.9}$$

From (3.6) and (3.7) it is not difficult to obtain an expression for the shear stress

$$\tau = -\langle \rho v_r' v_\varphi'\rangle = \rho l^2 \left|\frac{d\langle v_\varphi\rangle}{dr} - \frac{\langle v_\varphi\rangle}{r}\right| \left(\frac{d\langle v_\varphi\rangle}{dr} - \frac{\langle v_\varphi\rangle}{r}\right) = \rho\varepsilon \left(\frac{d\langle v_\varphi\rangle}{dr} - \frac{\langle v_\varphi\rangle}{r}\right) \tag{3.10}$$

which for $< v_\varphi > /r \ll d< v_\varphi > /dr$ (at the wall) reduces to the previously obtained formula (2.9).

As in [10], we assume that in fully developed annular turbulent flow, the scale of turbulence l is proportional to the radius

$$l = \alpha r, \qquad \alpha = \frac{v_{*1}}{2c_0 v_1} \tag{3.11}$$

Here, the constant α is determined from relations (3.1), (3.9), and (3.10).

Making use of relations (3.7), (3.11), we write formulas (3.3), (3.8) in final form:

$$v_{\varphi i}{}^* = k^{1/2} c^{-1/6} \frac{1}{r/r_i} \tag{3.12}$$

$$v_{ri}{}^* = \left[kc^{1/3} - \frac{2c}{k}\right]^{1/2} \frac{c^{-1/3}}{r/r_i} \tag{3.13}$$

$$v_{xi}{}^* = \left[2 - 2kc^{1/3} + \frac{2c}{k}\right]^{1/2} \frac{c^{-1/3}}{r/r_i} \tag{3.14}$$

$$v_i{}^{**} = \frac{1}{(r/r_i)^2} \tag{3.15}$$

$$E^* = \frac{c^{-1/3}}{r/r_i} \tag{3.16}$$

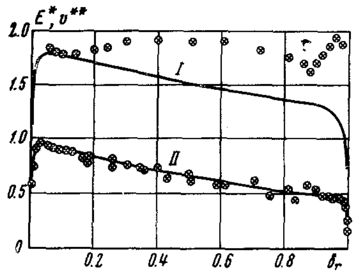

Fig. 4. The distribution of total turbulent energy (curve I) and the correlation (curve II) over the channel cross section.

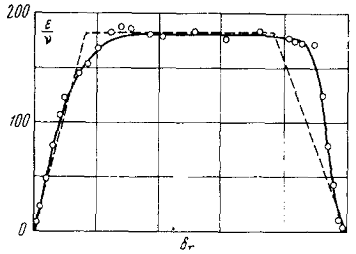

Fig. 5. Distribution of the turbulent viscosity coefficient over the channel cross section: solid line – experiment; dashed line – calculations.

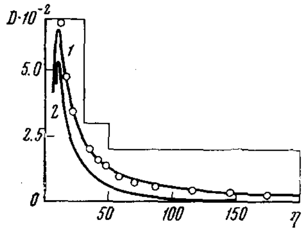

Fig. 6. Distribution of the pulsating-energy-balance components in the flow region at the rotating wall. The solid lines are theoretical curves; (1) generation – dissipation; (2) contribution of the action of viscosity on large-scale pulsating motion to dissipation.

We determine the turbulent viscosity coefficient ε from Eq. (3.10) by substituting into it the values of l and $< v_\varphi >$ from equalities (3.11) and (3.1):

$$\frac{\varepsilon}{\nu}=\frac{1}{2c_0}\frac{R^+}{v_1/v_{*1}} \qquad \left(R^+=\frac{v_{*1}r_1}{\nu}=\frac{v_{*2}r_2}{\nu}\right) \tag{3.17}$$

Integrating Eq. (3.9) with allowance for relation (3.11), we obtain the velocity distribution in the turbulent flow core

$$\frac{\varphi}{R^+ + (-1)^{i+1}\eta_i}=\frac{\varphi^*}{R^+ + (-1)^{i+1}\eta_i^*}-c_0R_0\left\{\frac{1}{[R^+-(-1)^{i+1}\eta_i^*]^2}-\frac{1}{[R^+-(-1)^{i+1}\eta_i]^2}\right\} \tag{3.18}$$

where $R_0 = v_1 r_1/\nu$; $\eta_i^* = y^* v_{*i}/\nu$ is the value of the generalized coordinate at the boundary layer interface; and $r^* = r_i + (-1)^{i+1} y^*$, $(i = 1, 2)$: φ^* is the velocity at this interface.

4. Comparison with Experiment. In Figs. 2-6, the computational relations are compared with results of experimental studies of the hydrodynamics of turbulent annular flow in channels with an inner rotating cylinder [5, 11].

Figure 2 shows the universal velocity profile φ plotted from formulas (2.6) and (3.18), which correlates well with experiment. As in [8], the separation of the flow into a laminar sublayer, a transition region and a turbulent core derives directly from the initial system of equations, without any special assumptions.

The distribution over the channel cross section, of the pulsation intensity of the components of the velocity vector v_{ij}^*, the total turbulent energy E^*, the correlation v_i^{**}, and the turbulent viscosity coefficient ε/ν, obtained from formulas (2.1), (2.8), and (3.12), (3.17), are also in good agreement with experiment (Figs. 3, 5).

Figure 6 shows the distribution of the pulsating-energy-balance components at the rotating wall. The same pattern occurs at the stationary wall. In addition to the good correlation between the theoretical curve and the experiments, the close conformity between the distribution pattern obtained and the corresponding pattern for rectilinear flows is noteworthy.

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